ANALYSIS SEMINAR

COMPACT FAMILIES OF SETS

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Abstract:

A topological space X is said to be compact if every filter base on X has a cluster point (Vietoris, 1920) or, equivalently, every open cover of X has a finite subcover (Alexandrov and Urysohn, 1922).

Many other notions of compactness arise in topology and analysis. To name a few, people investigate spaces that are countably compact, sequentially compact, Lindelöf (this also is a compactness type property), paracompact, metacompact, Eberlein compact, angelic, pseudocompact, feebly compact and on....and on...

If $K \subset X$, then K is a compact (in any sense) subset of X whenever K as a subspace of X is compact. Let now $\mathcal{K}$ be family of subsets of X. What it would mean that $\mathcal{K}$ is compact? Can we have a common principle that covers (at least a fair number of) the definitions given above and applies to the families of subsets? It looks that I now know the answers.

I will discuss the notions of $\mathbb{P}/\mathbb{R}$-compact (midcompact, ultracompact) at A family $\mathcal{B}$ of sets and the role of filter D-compactness as a unifying scheme.