

# Bregman distance, approximate compactness and Chebyshev sets in Banach spaces

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Abstract:

In this paper, we first introduce the notion of locally uniformly totally convex functions defined on a Banach space and discuss its relations to totally convex, essentially strictly convex, and uniformly convex functions. We then present some sufficient conditions for the (norm-weak) upper semicontinuity and the (norm-weak) continuity of the Bregman projection operator  $P_g$  and the relative projection operator  $P_{g,C}$  in terms of the notion of  $D$ -approximate (weak) compactness whenever  $g$  is either locally uniformly totally convex function or coercive, and  $C$  is a nonempty closed subset of  $\text{int}(\text{dom}g)$ . We finally present certain sufficient conditions as well as equivalent conditions for the convexity of a Chebyshev (in the sense of Bregman distance) subset of a Banach space  $X$ . Our results extends the corresponding results of [Bauschke, et al., J. Approx. Theory, doi:10.1016/j.jat.2008.08.014] to infinite dimensional spaces.