Number Theory Seminar

Wednesday, October 26 2022
4:00 pm in Hume 321

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Generalized arithmetic progressions and Diophantine approximation by polynomials

ABSTRACT

We discuss two related notions of "approximate subgroups" inside finite sets of integers: Bohr sets, which capture simultaneous diophantine approximation, and symmetric generalized arithmetic progressions (GAPs). For example, fix $N, d \in \mathbb{N}$ and consider the following pair of questions:

1. For fixed $\alpha_1, \dots, \alpha_d \in \mathbb{R}$, how small can we make $\max\{\|n^2\alpha_1\|_{\mathbb{T}}, \dots, \|n^2\alpha_d\|_{\mathbb{T}}\}$ with $1 \leq n \leq N$, where $\|\cdot\|_{\mathbb{T}}$ is distance to the nearest integer?

2. How large can a set of the form $\{x_1\ell_1 + \cdots + x_d\ell_d : -L_i \leq \ell_i \leq L_i\} \subseteq [-N, N]$ be before it is guaranteed to contain a perfect square?

Our discussions range from classical facts like the Kronecker approximation theorem and Linnik's theorem, to a recent breakthrough result of Maynard and its potential future applications. In between we survey results including previous joint work with Lyall and Croot.