Number Theory Seminar

Friday, October 20th, 2023

3:00-4:00pm in Hume 321

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A Generalization of the Grunwald-Wang Theorem

ABSTRACT

The Grunwald-Wang theorem for n^{th} powers states that a rational number a is a n^{th} power in $\mathbb{Q}_p$ for almost every prime p if and only if either a is a perfect n^{th} power in rationals or $8 \mid n$ and $a = 2^{\frac{n}{2}} \cdot b^n$ for some rational b. In this talk, we will present a generalization of a Grunwald-Wang theorem, from a single integer a to a subset A of rational numbers. More specifically, let q be the smallest prime dividing the natural number $n \geq 2$. A finite subset A of rationals with cardinality $\leq q$ contains a n^{th} power in $\mathbb{Q}_p$ for almost every prime p if and only if either A contains a n^{th} power in rationals or n is even and A is two-element subset of certain form. If time permits, we will also show that our generalization is optimal, i.e., for every $n \geq 2$, there are infinitely many subsets A of rationals of cardinality $q + 1$ that contain a n^{th} power in $\mathbb{Q}_p$ for almost every prime p but neither contain a perfect n^{th} power in rational nor contain a 2-element subsets of the above kind when n is even.