Combinatorics Seminar

Friday, December 1st, 2023

11:00am-12:00pm in Hume 321

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On the coequal values of total chromatic number and chromatic index

Abstract

The *chromatic index $\chi'(G)$* of a graph G is the least number of colors assigned to the edges of G such that no two adjacent edges receive the same color. The *total chromatic number $\chi''(G)$* of a graph G is the least number of colors assigned to the edges and vertices such that no two adjacent edges receive the same color, no two adjacent vertices receive the same color and no edge has the same color as its two endpoints. The chromatic index and the total coloring number are two of few fundamental graph parameters, and their correlation has always been a subject of intensive study in graph theory.

By definition, $\chi'(G) \leq \chi''(G)$ for every graph G. In 1984, Goldberg conjectured that for any multigraph G, if $\chi'(G) \geq \Delta(G) + 3$ then $\chi''(G) = \chi'(G)$. The conjecture is shown asymptotically true. More specifically, for a multigraph G with maximum degree Δ sufficiently large, $\chi''(G) = \chi'(G)$ provided $\chi'(G) \geq \Delta + 10\Delta^{35/36}$. When $\chi'(G) \geq \Delta(G)+2$, the chromatic index $\chi'(G)$ is completely determined by the fractional chromatic index. Consequently, the total chromatic number $\chi''(G)$ can be computed in polynomial-time in this case. In this talk, I will discuss the recent progress and the proof techniques.