Combinatorics Seminar

Wednesday, March 27th, 2024

2:00-3:00pm in Hume 321

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An Ore-type condition for hamiltonicity in tough graphs and the extremal examples

ABSTRACT

Let G be a t-tough graph on $n \geq 3$ vertices for some $t > 0$. It was shown by Bauer et al. in 1995 that if the minimum degree of G is greater than $\frac{n}{t+1} - 1$, then G is hamiltonian. In terms of Ore-type hamiltonicity conditions, the problem was only studied when t is between 1 and 2, and recently the second author proved a general result. The result states that if the degree sum of any two nonadjacent vertices of G is greater than $\frac{2n}{t+1} + t - 2$, then G is hamiltonian. It was conjectured in the same paper that the "$+t$" in the bound $\frac{2n}{t+1} + t - 2$ can be removed. Here we confirm the conjecture. The result generalizes the result by Bauer, Broersma, van den Heuvel, and Veldman. Furthermore, we characterize all t-tough graphs G on $n \geq 3$ vertices for which $\sigma_2(G) = \frac{2n}{t+1} - 2$ but G is non-hamiltonian. This is joint work with Masahiro Sanka.