Combinatorics Seminar

Wednesday, April 9th, 2025
4:00-5:00 pm in Hume 331

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A generalization of Sárközy's theorem in function fields

Abstract

Sárközy's theorem says that if $A \subset \mathbb{Z}$ has positive upper asymptotic density, then there are distinct $a_1, a_2 \in A$ and $n \in \mathbb{Z}$ such that $a_1 - a_2 = n^2$. The same is true if n^2 is replaced by $F(n)$ for any polynomial $F \in \mathbb{Z}[x]$ with constant term zero. Using the Croot–Lev–Pach polynomial method, Green proved an $\mathbb{F}_q[t]$-analog of Sárközy's theorem with strong quantitative bounds, but required a technical condition on the number of roots of the polynomial $F \in \mathbb{F}_q[t][x]$. This condition was recently removed by Li and Sauerman. We generalize Green's argument to accommodate equations in more variables in $\mathbb{F}_q[t]$, while pointing out that the technical condition can be removed by means of a simple observation.