

Number Theory Seminar

Monday, April 7, 2025

2:00 pm in Hume 321

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Exponential sums weighted by additive functions and their associated partition asymptotics

ABSTRACT

We introduce a general class \mathcal{F}_0 of additive functions f such that $f(p) = 1$ and prove a tight bound for exponential sums of the form $\sum_{n \leq x} f(n)e(\alpha n)$ where $f \in \mathcal{F}_0$ and $e(\theta) = \exp(2\pi i\theta)$. Both ω , the number of distinct primes of n , and Ω , the total number primes of n , are members of \mathcal{F}_0 . As an application of the exponential sum result, we use the Hardy-Littlewood circle method to find the asymptotics of the Goldbach-Vinogradov ternary problem associated to Ω , namely we show the behavior of $r_\Omega(N) = \sum_{n_1+n_2+n_3=N} \Omega(n_1)\Omega(n_2)\Omega(n_3)$, as $N \rightarrow \infty$. Lastly, we end with a discussion of further applications of the main result.