

Combinatorics Seminar

Wednesday, April 16th, 2025

4:00-5:00 pm in Hume 321

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Independent transversals in multipartite graphs

Abstract

An independent transversal in a multipartite graph is an independent set that intersects each part in exactly one vertex. We show that for any even integer $r > 0$, there exist $c > 0$ and n_0 such that every r -partite graph with parts of size $n > n_0$ and maximum degree less than $rn/(2r - 2)$ contains at least cn^{r-1} independent transversals. This is best possible up to the value of c and confirmed a conjecture of Haxell and Szabo from 2006 and partially answered a question raised by Erdős from 1972.

We also show that for integer $s > 1$, even integer $r > 0$ there exist $c' > 0$ and n_0 such that every r -partite graph with parts of size $n > n_0$ and maximum degree less than $rn/(2r - 2) - cn^{1-1/s}$ contains an independent set with exactly s vertices in each part. This is best possible up to the value of c for $s = 2, 3$ due to known constructions for the Zarankiewicz problem.

This is a joint work with Yantao Tang.